SUPPLY-VOLTAGE OPTIMIZATION BY THE FREQUENCY CONTROL OF HIGH-POWER INDUCTION MOTORS

Abstract: Analyzed is the PMW supply voltage of high-power induction motors. An optimization problem is defined for the determination of the switching-angles set with minimization of the voltage distortion. By means of own software packages, the sets of switching angels of a number 4 to 14 within a half-period are determined. The results are analyzed on the basis of the flux-linkage vector and compared to results of other authors.

Keywords: PMW supply voltage, switching angles, voltage distortion, induction motor, flux-linkage vector.

INTRODUCTION

The Pulse-width Modulation (PWM) to obtain a voltage with variable both effective value and frequency is applied for supplying induction motors within the entire power range. The vector-control methods are widely employed for control induction motors of low and medium power. An important constrain by the electrical drive with high-power (>500 kW) motors is a minimum number of switching . A preliminary determination of the set of switching angles during the supply-voltage period appears an obligatory step in the frequency-inverter design ensuring the PWM-based supply voltage with a minimum switching number and a reasonable goodness factor.

CONSIDERATION OF THE ELECTRICAL CIRCUIT

The frequency inverter is supplied by a three-phase line voltage U. A three-phase uncontrollable bridge inverter is used and the output pulsed voltage

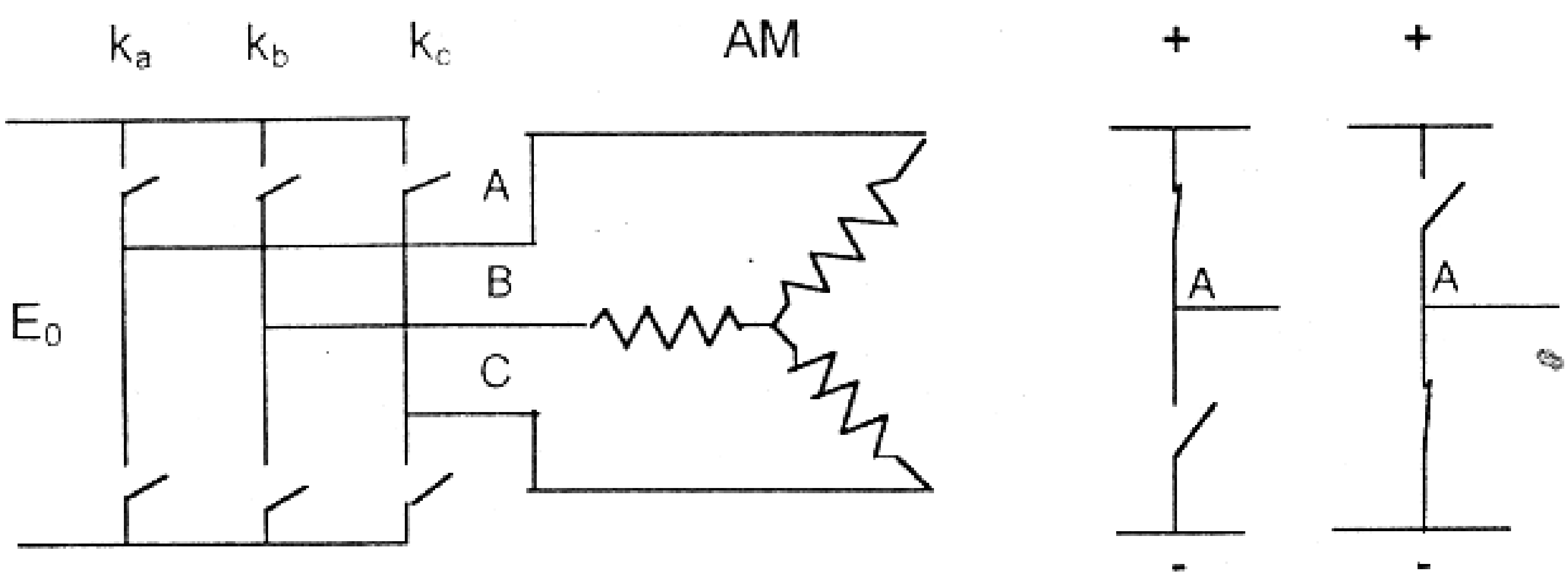

Fig.1. Elecrical circuit and the possible switching of phase A

being smoothed by condensers. The maximum value of this voltage is equal to the maximum value of the input line voltage $\sqrt{2}U$ and its mean value is $E_0 = \frac{3}{\pi} U_{max} \approx 0.955 U_{max} \approx 1.35U$. The following consideration are made for a smoothed direct voltage E_0. This voltage feeds the three phases of the motor AM (Fig.1). Thereby, each phase can be connected to the plus or minus of the direct voltage. In a steady state, one of the two switches of each phase is "in", the variants with both switches simultaneously "in" or "out" do not occur. Thus, for the three-phase system 8 combinations (6 activ and 2 passiv ones) are possible. By defining of activ-combination order **s** and phase-switch value 1 (plus-connected) or 0 (minus-connected) all the combinations can be presented by Table 1.

Table 1

s	0	1	2	3	4	5	-	-
k_a	1	1	0	0	0	1	0	1
k_b	0	1	1	1	0	0	0	1
k_c	0	0	0	1	1	1	0	1

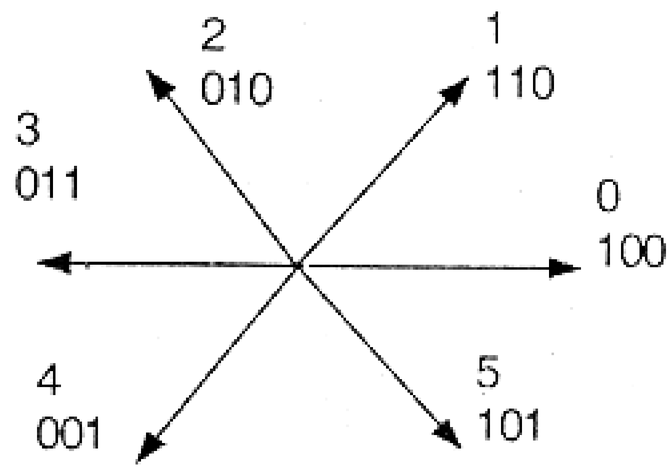

Fig.2. The 6 possible active values of the voltage vector U.

The phase voltages relative to the negative pole can be expressed as: $U_a = k_a \cdot E_0$; $U_b = k_b \cdot E_0$; $U_c = k_c \cdot E_0$. The supply -voltage vector is defined as $\vec{U} = \frac{2}{3} E_0 e^{js\pi/3}$. Its possible 6 active values are displayed in Fig.2. For the two passive combination (000 and 111) the voltage is zero. By defining the phase voltages relative to the direct-voltage-medium point the phase-A voltage (general case) is shown in Fig.3.

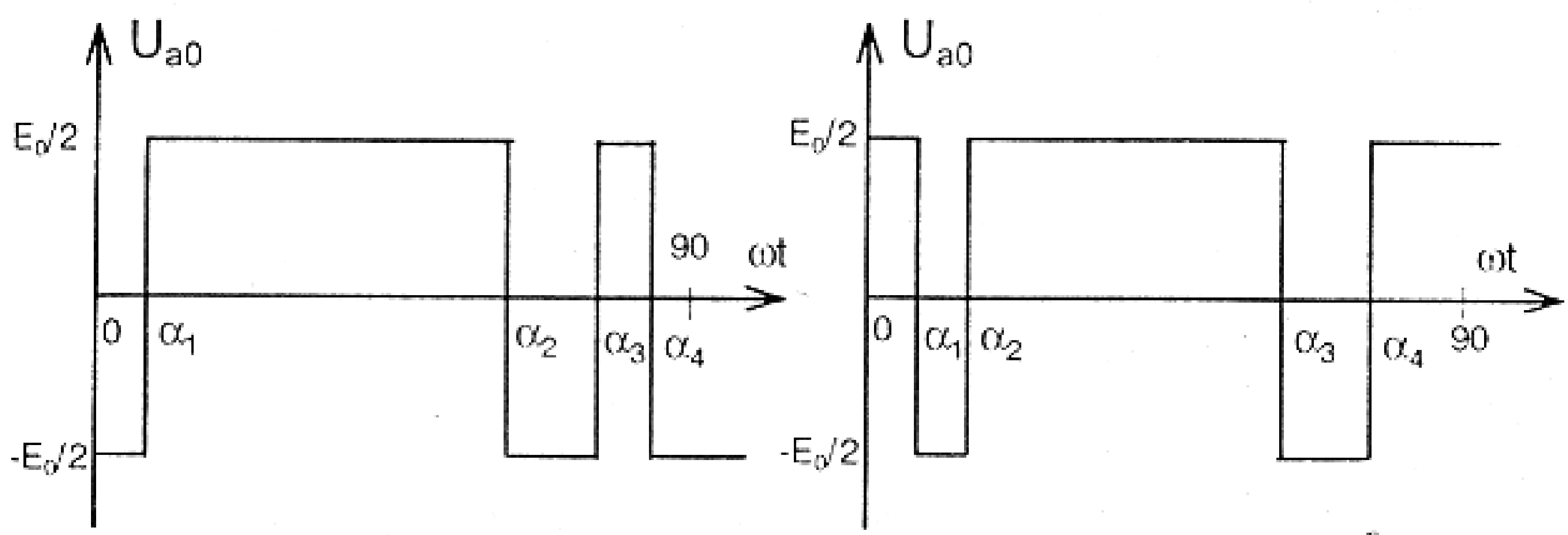

Fig.3 The phase voltage U_{a0} relative to the direct-voltage medium

There are one obligatory switching at the beginning of each half-period and N switching angles during a quarter of a period (in Fig.3, N=4). In the case of a symmetric three-phase power supply, the total switching number for the three phases during one period amounts to $N_p=6(2N + 1)$. Two cases are possible arbitrarily labeled with $Z_n=-1$ when the voltage starts with a negative pulse (l.h.s. of Fig.3) and with $Z_n=+1$ with a positive pulse (r.h.s. of Fig.3). The voltages of the other two phases have the same shape as U_{a0} but are shifted by 120°.

The line supply voltages result from the difference of the phase voltages: $U_{ab}=U_{ao}-U_{b0}$. For the phase voltages U_{a0} and U_{bo} in the case of $Z_n=-1$, the line voltage U_{ab} is given in Fig.4. It is shifted by 60° relative to U_{a0}. During a half period, 2.N+1 pulses in the line voltage result from the 2.N+1 switching. A definition of the motor phase voltage is somewhat arbitrary since in a star connection with an isolated star point the star-point potential is not definitely zero. Nevertheless, a phase voltage is arbitrarily defined neglecting the

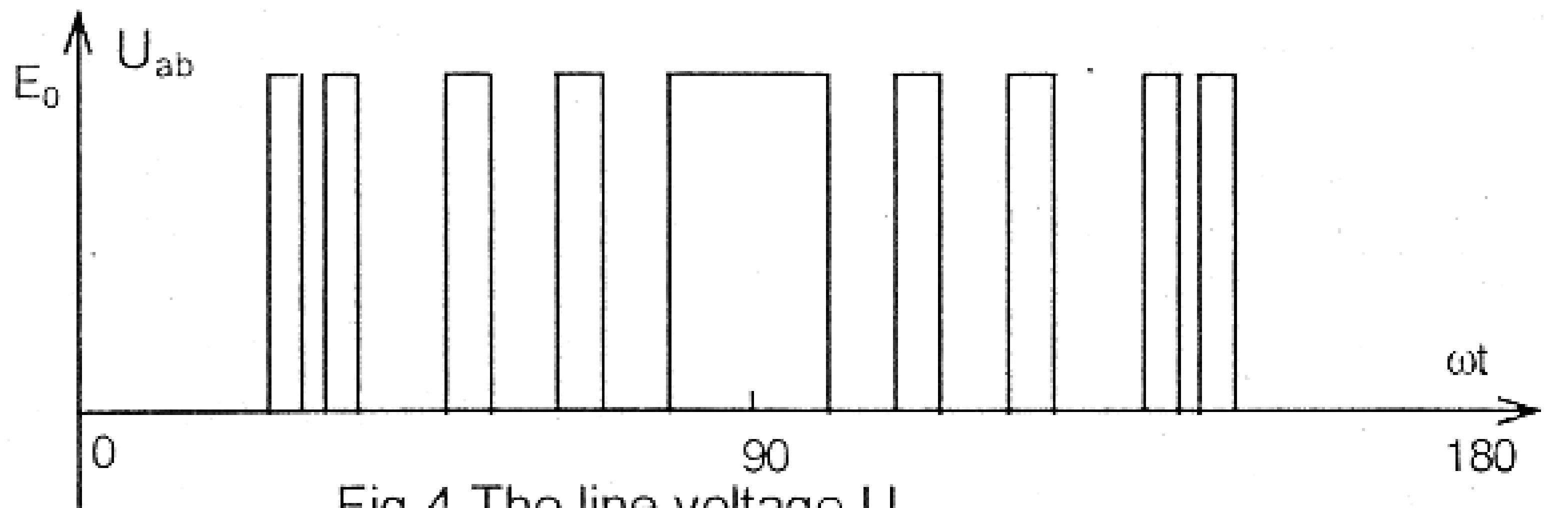

Fig.4 The line voltage U_{ab}

transient processes and assuming that the star-point potential changes by jumping at the switching moment.

Then following relations are valid: $U_A = \frac{U_{ab} - U_{ca}}{3}$; $U_B = \frac{U_{bc} - U_{ab}}{3}$; $U_C = \frac{U_{ca} - U_{bc}}{3}$.

This arbitrary phase voltage is presented in Fig.5.

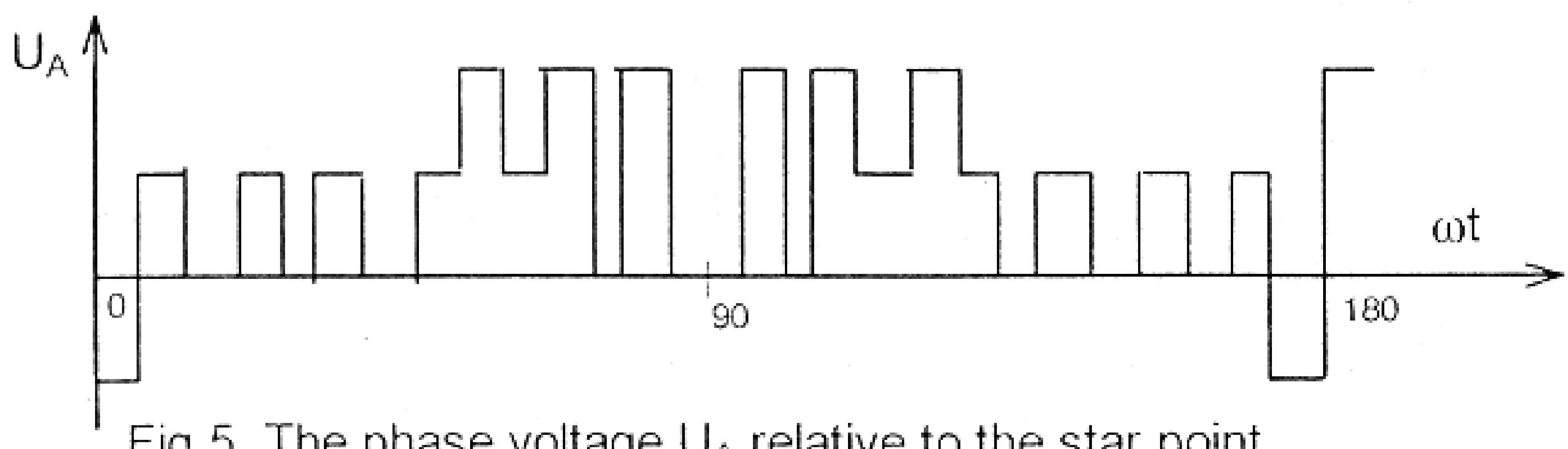

Fig.5. The phase voltage U_A relative to the star point

The voltage comes out at two levels. This means that the phase-A voltage depends on its own switch (0 or 1), but also on the position of the switches of the other two phases. One may describe the star-point potential in a similar way.

MATHEMATICAL ANALYSIS

Analyzed is the voltage U_{ao} presented in Fig.3. It is an explicit function of the switching angles whereas the angular dependence of the other voltages is an implicit complicated function. In Fig.6, the voltage U_{a0} is displayed in the general case of N switching angles and Z_n=-1.

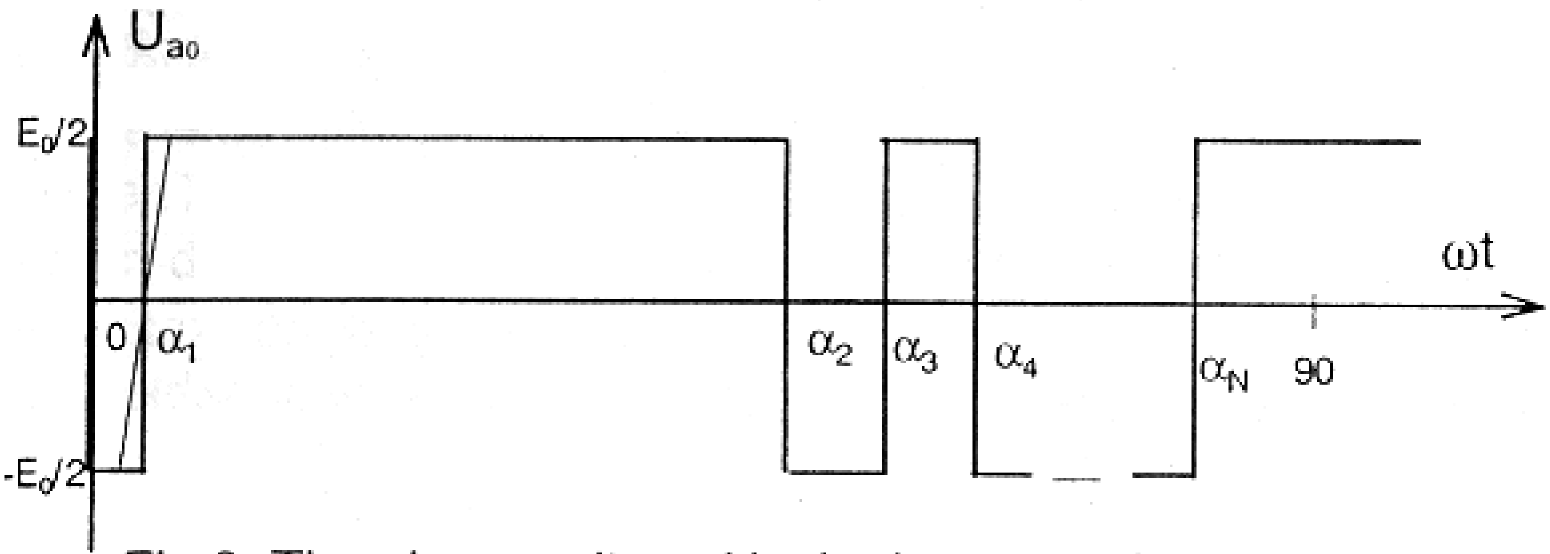

Fig.6 .The phase voltage U_{a0} in the case of N switching

One may consider the switching (i) in the ideal case of potential jump (solid line), or (ii) taking into account the transient processes in the thyristors with a possible linear approximation (dashed line). The function $U_{ao}(\omega t)$ is decomposed in a Fourier series, this function is odd and symetric with respect to 90°. In the series, only the odd sine coefficients are included. The connection of the stator winding is of star type, with an isolated star point. This allows the neglecting of higher harmonics of an order divisible by 3. By substitution of ωt with α the function is expressed by the relations :

$$U_{ao}(\alpha) = \sum_{k=1,5}^{\infty} U_{maxk} . \sin(k\alpha) \tag{1}$$

$$U_{maxk} = \frac{4}{\pi} \int_0^{\pi/2} U_{a0}(\alpha) . \sin(k\alpha) d\alpha \tag{2}$$

where U_{maxk} is the maximal value of the k-th voltage harmonic. After integration for the ideal case of switching (i) in Eq.2 it follows :

$$U_{maxk} = \frac{2}{\pi} \frac{E_0 Z_n}{k} \left[1 + 2\sum_{i=1}^{N} (-1)^i \cos(k\,\alpha_i) \right] \tag{3}$$

where i is the switching-angle order.

For a given steady state characterized by the frequency and the effective value of the fundamental, one should know the set of N switching angles which define a supply voltage with a given U_1(the effective fundamental value) and with small high harmonics. In electrical machines, one defines

$$\sigma = \sqrt{\sum_{k=2}^{\infty} \left(U^{k}\!/_{k} \right)^2} \tag{4}$$

In the general case, the optimization problem reads:

- Determine the set of switching angles $\alpha_1, \alpha_2, \ldots, \alpha_N$ at given N and Z_n ;

- Minimize the function σ (Eq.4) at the limiting conditions

$$0° < \alpha_1 < \alpha_2 \ldots < \alpha_N < 90° \quad (5)$$

$$U_1 = \frac{U_{max1}}{\sqrt{2}} = \frac{2}{\pi}\frac{E_0 Z_n}{\sqrt{2}}\left[1 + 2\sum_{i=1}^{N}(-1)^i \cos(\alpha_i)\right] \quad (6)$$

The optimization problem poses serious difficulties since the function σ is a sum of trigonometric functions and contains numerous local extrema. One correct constrain would be to perform the summation up to a given maximum number k_{max}. However, the attempt for optimization with different software packages in this way did not deliver satisfactory results. Therefore, the function σ was analyzed in the whole range and allowed sub-ranges were defined. Then, a localization of the searched values and their refinement can be achieved. In order to reduce the computing to a reasonable number of steps following assumptions are made:

- The optimization function is $\varphi = \left(\frac{\sigma\sqrt{2}\pi}{2E_0}\right)^2 = \sum_{k=5,7}^{49}\left(\frac{B_k}{k}\right)^2$,

  where $B_k = \frac{U_k\sqrt{2}\pi}{2E_0}$ is the normalized value of the amplitude of the k-th harmonic;
- The angles $\alpha_1, \alpha_2, \ldots, \alpha_{N-1}$ are being stepwise chosen;
- The last angles α_N is determined from the limiting condition (6)

$$\alpha_N = \arccos\frac{A - \sum_{i=1}^{N-1}(-1)^i \cos\alpha_i}{(-1)^N}$$

  where A is the constant $A = \frac{1}{2}\left(\frac{\sqrt{2}\pi U_1}{2E_0 Z_n} - 1\right)$; the angle α_N must satisfy the condition (5).

RESULTS

The calculation performed for given N and B_1 enable important conclusions:

A) Combinations with both Z_n=-1 and Z_n=+1 appear possible.

B) The switching angles turn out to be irregularly distributed. If the full range of 0°-90° is divided into three equal zones : 0°-30°, 30°-60° and 60°-90°, the switching angles falls into the first and third zones. In the second zone (30°-60°), no switching angles occur.

C) At small values of the fundamental B_1, the switching angles falls into the third zone. With the increase of the value of B_1 the angles "move" to the first zone.

In Fig.7, the optimization results at N=4 for all possible sets of angles are presented. On the l.h.s., the sets of the four angles at Z_n=+1 are illustrated with the voltage U_{a0} starting with a positive pulse. Set **a**[0+4], **b**[2+2], and **c**[4+0] ([x+y] are the number of angles x in the zone 0°-30° and y at 60°-90°)

are of interest in the chosen regions of B_1. On the r.h.s. of Fig.7 with the

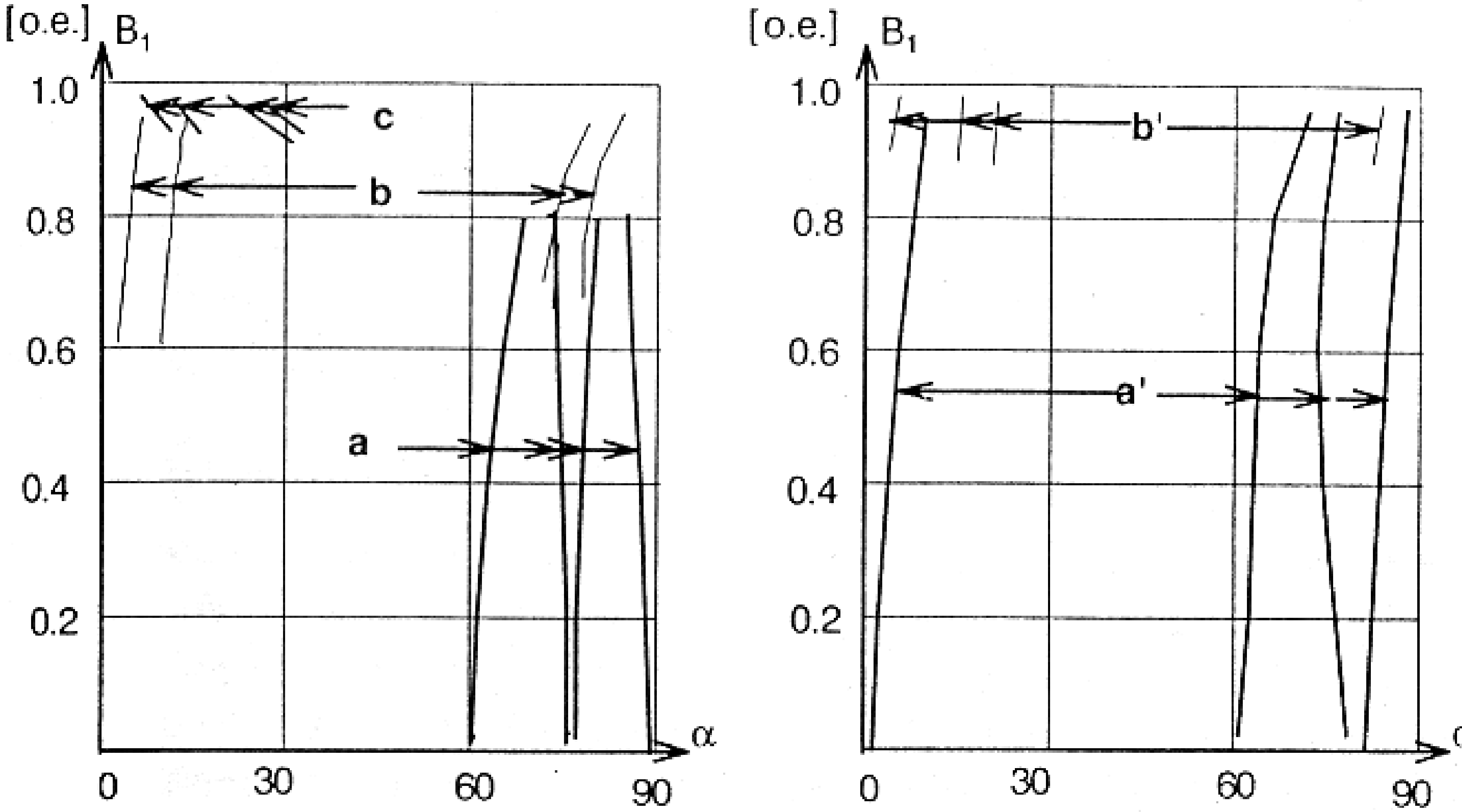

Fig.7 shows the results at N=4 for all possible sets of switching angles

angles at Z_n=-1 are the sets **a'** and **b'**.

By means of the minimized values of the function φ as shown in Fig.8 the regions of the different set can be defined. At the small values of B_1 the operational set is **a** (Z_n=+1, 4 angles in the zone 60°-90°). With the increase of B_1 the set a, a', b, b', c consecutively appear. The angle values found correspond to a high-quality voltage, i.e. with minimum voltage distortion, the content of high-harmonic being bellow 2 %.

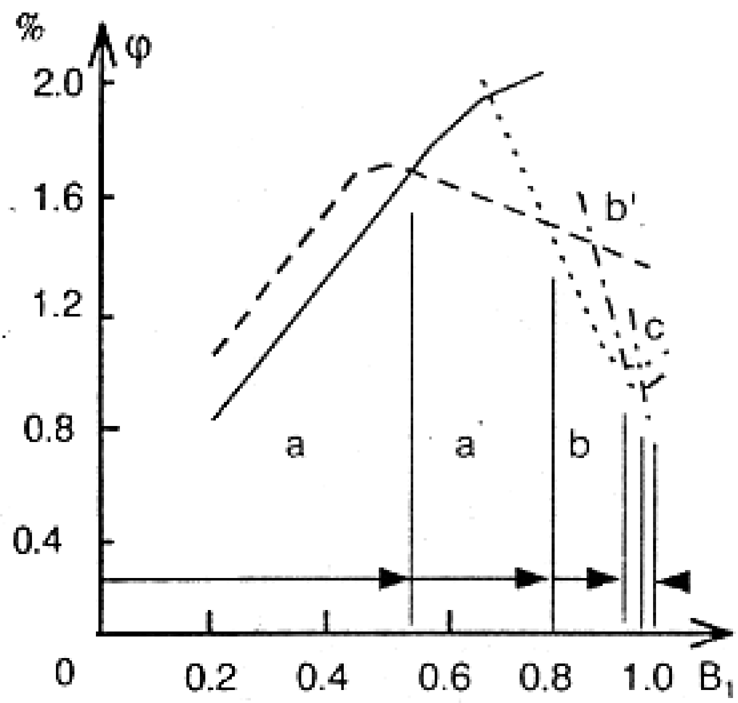

Fig.8. The voltage distortion

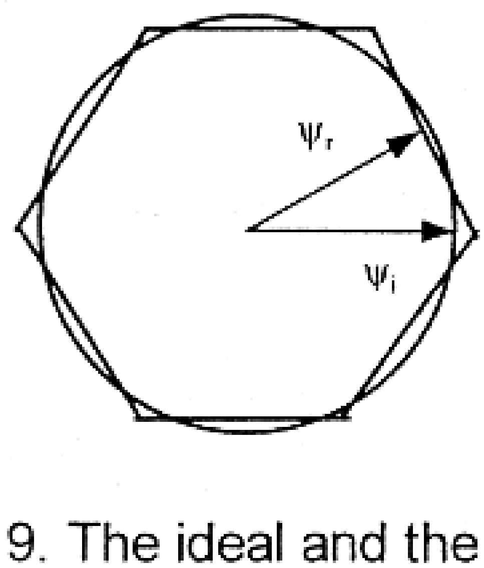

Fig.9. The ideal and the real vector locus

Further optimization calculations are performed for N=5÷14. The results are the same as those for N=4. With increasing N the values of the function φ decrease.

QUALITATIVE ANALYSIS

One possible way to qualitatively check the goodness of the optimization procedure described above is the analysis of the flux-linkage vector. If we neglect the active voltage loss in the stator equation of the asynchron motor $\bar{u} = r.\bar{i} + \frac{d\bar{\psi}}{dt}$ what is well justified in case of the high-power motors considered, this equation reduces to $\bar{u} \approx \frac{d\bar{\Psi}}{dt}$. Since the voltage vector $\bar{u}$ is defined for each time interval, one may evaluate the change of the flux-linkage vector $\bar{\psi}$. By a sine-voltage supply, the peak of $\bar{\psi}$ describes a circle. At PMW, the voltage has 6 active and 2 passive combinations (Fig.3). If the switching is merely at the beginning of the half period (N=0), the vector locus is a hexagon (Fig.9) since every 60° the voltage vector takes consecutively all 6 active combinations (***s***=0,...,5). At number of switching angles N>0, the hexagon is modified, coming in the optimal situation close to the ideal case, a circle.

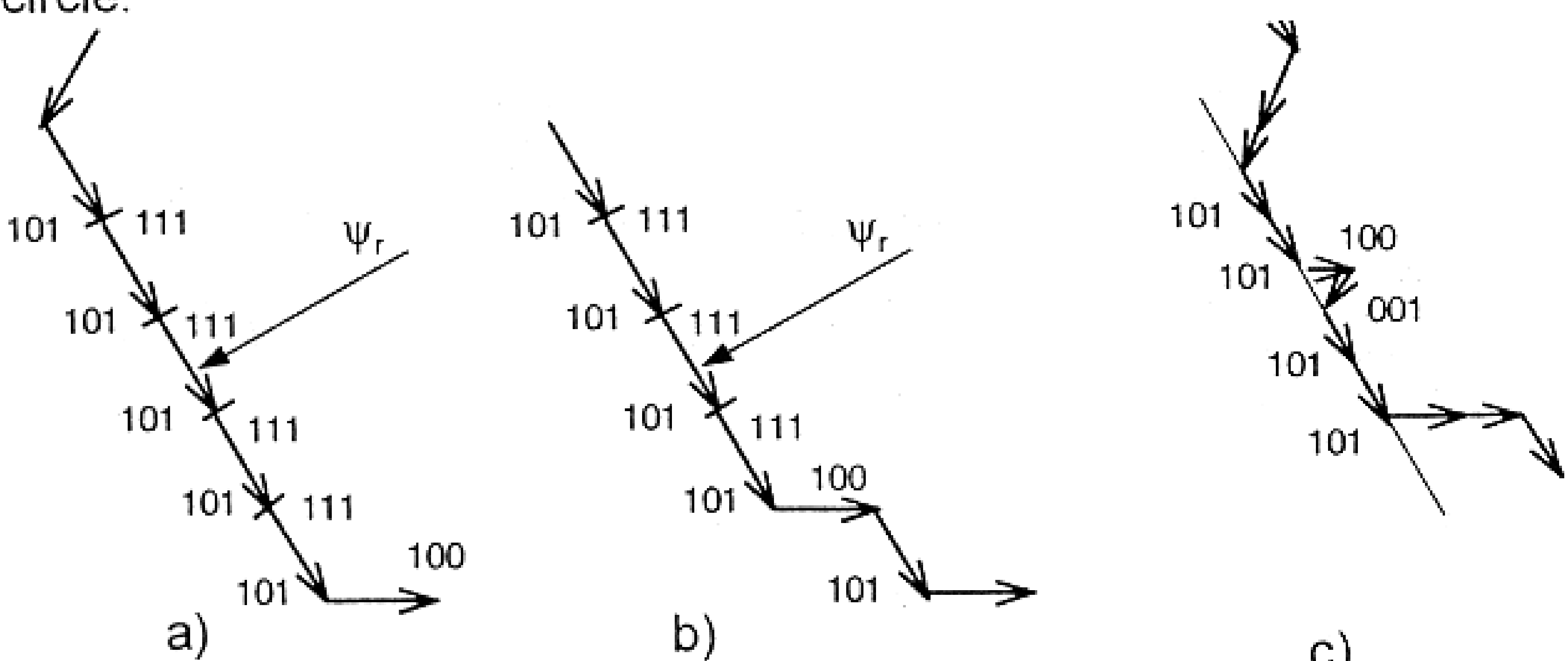

Fig.10. The modified hexagon in three cases

The calculated sets of angles result in a modified hexagon shown in Fig.10a, b. The first case is at N=4 (set **a** in Fig.7, the angles are in the third zone) and corresponds to an unmodified hexagon with pauses (at combination 111, $\bar{u}$=0). The second case is at N=4 (set **a'** in Fig.7, angles:1+3) and corresponds to a modified hexagon with smoothed apices and with pauses.

The analysis of the supply voltage confirms the good quality of the numerical optimization procedure outlined above.

Further, an optimization calculation with other function is performed: to search for minimum electrical rotor losses. The results are identical: the angles are situated qualitatively in the same way in the above mentioned zones and the quantitative differences in the angles found do not exceed 1°.

A comparison is made with results of other authors [3, 4]. Considering the same optimization function (Eq.4), they used powerful mathematical methods: Lagrange multipliers, Newton's iterative method for approximate solution of equation systems. Their results practically coincide with the present ones for a small number of switching angles ($N<7$) but disagree at larger N. For $N>7$, in their work switching angles appear in the second zone (30°-60°) as well. The analysis of those results by means of the flux-linkage vector reveals a hexagon with the presence of distorted zones, "kinks". In Ref.[3], up to $N=6$ the results obey the regularities found in this work and the calculated angles are situated in the first and third zones. For $N\geq7$, however some calculated angles are in the second zone. The relevant flux-linkage vector analysis point in that case at an unsatisfactorily modified hexagon. Fig.10c illustrates the angles calculated [3] for $N=10$, $B_1=0.24$, $Z_n=+1$ and reveals a "kink". This is a clear signal that the optimization procedure has reached a local minimum, missing the global minimum. One possible reason for this discrepancy is that often the minimum of the function can be in a very narrow interval, thus requiring an angle determination with an accuracy of 0.001°.

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